Continuous measurements allow predicting the future and retrodicting the past of quantum systems. These two possibilities are not exhaustive, and some measurements leave the future and the past uncertain, yet establish a relation between them; such measurements are non-local in time. We introduce a procedure for continuous time-non-local measurements formulated as a superposition of prediction and retrodiction, and apply it to the problems of teleportation and conditional state transfer between two quantum oscillators interacting with traveling fields.

The two observables that need to be measured to transfer a state are the position and momentum differences between the source oscillator at the initial time and the target oscillator at the final time of the interaction. Such measurements do not condition regular quantum states, but two-time states that contain components propagating in opposite directions in time.

Our approach enables us to analytically determine the fidelities of the state transfer based on homodyne detection, and to identify strategies for performing the transfer perfectly across a wide range of linear oscillator-field interactions beyond the pure beam-splitter and two-mode-squeezing types.